PHYSNET

MISN-0-484

FOURIER INTEGRALS: PART 1

Math Physics

Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

FOURIER INTEGRALS: PART 1

by
R. D. Young, Dept. of Physics, Illinois State Univ.

Title: **Fourier Integrals: Part 1**

Author: R. D. Young, Dept. of Physics, Illinois State Univ.

Version: 2/1/2000 Evaluation: Stage B0

Length: 2 hr; 8 pages

Input Skills:

1. Vocabulary: periodic function, trigonometric function, sine, cosine, Fourier expansion of $f(x)$, Parseval's identity, orthogonal functions, Kronecker's delta function.
2. Unknown: assume (MISN-0-483).

Output Skills (Knowledge):

K1. Define or explain each of the following terms or concepts: Fourier integral expansion $f(x)$, Fourier transform of $f(x)$, Fourier cosine transform and inverse cosine transform, inverse Fourier transform of $F(\omega)$, Fourier sine transform and inverse sine transform.

K2. Write down from memory Fourier's integral theorem.

Output Skills (Rule Application):

R1. Calculate Fourier transforms and inverse transforms when given the appropriate functions.

R2. Evaluate infinite integrals using Fourier integrals.

R3. Solve certain integral equations using Fourier transforms.

External Resources (Required):

1. G. Arfken, Mathematical Methods for Physicist, Academic Press (1995).
2. Schaum's Outline: Murray Spiegel, *Theory and Problems of Advanced Mathematics for Scientists and Engineers*, McGraw-Hill Book Co. (1971).

FOURIER INTEGRALS: PART 1

by

R. D. Young, Dept. of Physics, Illinois State Univ.

1. Introduction

In the last two units Fourier series were seen to be useful for representing certain functions in two cases:

(a) over an interval of the form $[0, L]$ or $[-L, L]$;

(b) over the infinite interval $(-\infty, \infty)$ *if the function is periodic.*

This unit introduces a way of representing a non-periodic function over $(-\infty, \infty)$ or $(0, \infty)$ using complex exponentials $e^{\imath\omega x}$ or sines or cosines. The technique employs the so-called Fourier transforms $f(x)$ and $F(\omega)$ where

$$f(x) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} d\omega \, F(\omega) e^{-\imath\omega x}$$

and

$$F(\omega) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} dx \, f(x) e^{+\imath\omega x} \, .$$

One of the most important uses of Fourier transforms is in expressing wave functions in quantum mechanics. The basic ideas involved are outlined in Sec. 15.6 *Momentum Representation* of Arfken. In MISN-0-486, we shall use the Fourier transforms to solve some practical problems in heat conduction.

2. Procedures

1. Read sections 15.2-15.4, of Arfken. Read pages 201-202 to end of section entitled Fourier Transforms, of Spiegel.

2. Underline in the text or write out the definitions and explanations of the terms and concepts of Output Skill K1 using an explanatory equation if necessary. One or two sentences should be sufficient.

3. Memorize and write from memory Fourier's Integral Theorem as stated on page 201 of Spiegel. Be sure to include the conditions on f(x) for the theorem to hold. You should also note that eqs. 15.17 and 15.20 of Arfken are equivalent to eqs. (3) and (4) in Chapter 8 of Spiegel, respectively.

4. Read through these Solved Problems in Spiegel:

   8.1 (Calculation of Fourier transforms and graphing)

   8.2 (Evaluation of infinite integrals using Fourier transforms)

   8.4 (Solving integral equations using Fourier transforms)

5. Solve these problems in the section on Supplementary Problems in Spiegel:

   8.16a (Fourier Transform)*

   8.16b (Evaluating infinite integrals using Fourier transforms)*

   8.19 (Solving an integral equation using Fourier transforms)

   Solve this problem in Arfken:

   15.3.9 (Fourier transform)

6. Cultural background procedure: In Arfken, read Sec. 15.1, on integral transforms in general and Sec. 15.6, on the use of Fourier transforms in quantum physics. This material is for general cultural background. You will not be tested on it, but it will come in handy someday.

* The answers on page 209 of Spiegel should be changed by an overall minus sign.

Acknowledgments

The author would like to thank Illinois State University for support in the construction of this lesson. Preparation of this module was supported in part by the National Science Foundation, Division of Science Education Development and Research, through Grant #SED 74-20088 to Michigan State University.