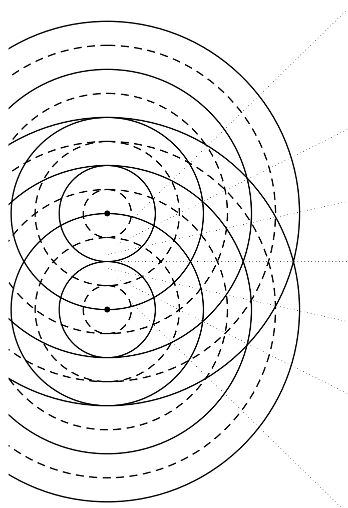
MISN-0-431

INTERFERENCE



Project PHYSNET Physics Bldg. Michigan State University East Lansing, MI

INTERFERENCE

by

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CONTENTS

Title: **Interference**

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Input Skills:

1. State the superposition principle for waves (MISN-0-430).

Output Skills (Knowledge):

K1. Vocabulary: coherent waves, incoherent waves, path difference.
K2. State the resultant intensity of two coherent waves.
K3. State the resultant intensity of two incoherent waves.
K4. State the conditions for maxima and minima of intensity for the interference of waves from two coherent sources in phase.
K5. Define the path difference for points far from two sources.
K6. State how a small hole is equivalent to a small source.
K7. Describe Young's two-slit interference experiment.

Output Skills (Problem Solving):

S1. Relate the amplitudes and intensities of two or more waves to the amplitude and intensity of the resultant wave when the individual waves sources are either coherent (in phase or 1/2 cycle out of phase) or incoherent.
S2. For two sources (or equivalent holes) emitting waves in phase or 1/2 cycle out of phase, relate the locations of the interference maxima or minima to the path difference, and thus also to the distance between the sources and to the wavelength (or frequency).

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INTERFERENCE

Abstract:
As discussed at the end of the preceding unit, the superposition principle for waves implies the existence of interference effects. In the present unit we shall study such interference effects in greater detail. As we shall see, the interference of waves leads to some remarkable observations, gives rise to important practical applications, and has far-reaching implications which will be explored in subsequent units.

SECT.

A INTERFERENCE AND PHASE DIFFERENCE

Consider two sinusoidal waves w_1 and w_2, of the same frequency, present at the same point. We now extend our discussion of text section F of Unit 430 by examining more closely how the interference between two such waves depends on the phases of these waves.

COHERENCE VERSUS INCOHERENCE

► *Coherent waves*

The waves may be produced in such a way that a definite relation between their phases is maintained at all times. (For example, this would be true for the sound waves produced by two loudspeakers operated from the same amplifier of a stereo set.) Such waves are called "coherent" in this sense:

Def. | **Coherent waves**: Waves whose phase difference remains always the same. | (A-1)

For example, if the maximum of one wave occurs always at the same instant as the maximum of the other wave, the waves are coherent and "in phase" (i.e., their phase difference remains equal to zero). Similarly, if the maximum of one wave always occurs at the same instant as the minimum of the other wave, the waves are coherent and half a cycle "out of phase" (i.e., their phase difference remains equal to one-half cycle).

► *Incoherent waves*

On the other hand, waves may be produced in such a way that nothing assures their synchronization. The phases of the waves may then vary relative to each other in some uncoordinated manner. Such waves are said to be "incoherent" in this sense:

Def. | **Incoherent waves**: Waves whose phase difference varies so that it is equally likely to assume any value. | (A-2)

For example, the light emitted from two different lamps (e.g., light bulbs) is *incoherent* since the individual atoms in the lamps emit light completely independently of each other. Hence the light wave emitted by one atom can have any phase relative to the light wave emitted by another atom.

Thus the maximum of one wave occurs sometimes at the same instant as the minimum of another wave.

► *Interference of Coherent Waves*

Let us first discuss the simple case of two *coherent* sinusoidal waves present at the same point, where one of these waves w_1 has an amplitude A_1 and the other wave w_2 has an amplitude A_2 (smaller or equal to A_1). By the superposition principle, the resultant wave at any instant is then simply equal to the sum $w = w_1 + w_2$ of the individual waves. How does the intensity I of this resultant wave depend on the phase difference between these waves?

► *Effective phase difference*

Because sinusoidal waves repeat themselves after every cycle, a phase difference of some integral number of cycles is equivalent to a phase difference of zero. (The maxima of both waves occur then also at the same instant.) Hence all observable results depend only on an "effective" phase difference between 0 and 1 cycle (disregarding any integral number of cycles). For example, a phase difference of 1.5 cycle is equivalent to an effective phase difference of 0.5 cycle.

► *Waves in phase*

Suppose that the two waves are "in phase" (i.e., that their effective phase difference is zero) so that the maximum of one wave occurs always at the same instant as the maximum of the other wave. (See Fig. A-1a and Fig. A-1b.) Then the amplitude A of the resultant wave has its maximum possible value: $A_{\max} = A_1 + A_2$. (See Fig. A-1c.) Correspondingly, the intensity I of the resultant wave has then also its maximum possible value $I_{\max}$, given by Relation (E-2) of Unit 430:

$$\boxed{I_{\max} = \gamma A_{\max}^2 = \gamma (A_1 + A_2)^2 \,.} \qquad \text{(A-3)}$$

Thus

$$I_{\max} = \gamma\,(A_1^2 + A_2^2 + 2A_1A_2),$$

or

$$I_{\max} = I_1 + I_2 + 2\gamma A_1 A_2, \qquad \text{(A-4)}$$

where $I_1 = \gamma A_1^2$ is the intensity of the first wave and $I_2 = \gamma A_2^2$ is the intensity of the second wave. This maximum possible resultant intensity is obviously larger than the sum of the individual intensities [by the last term in Eq. (A-4)] and corresponds to the most extreme case of constructive interference.

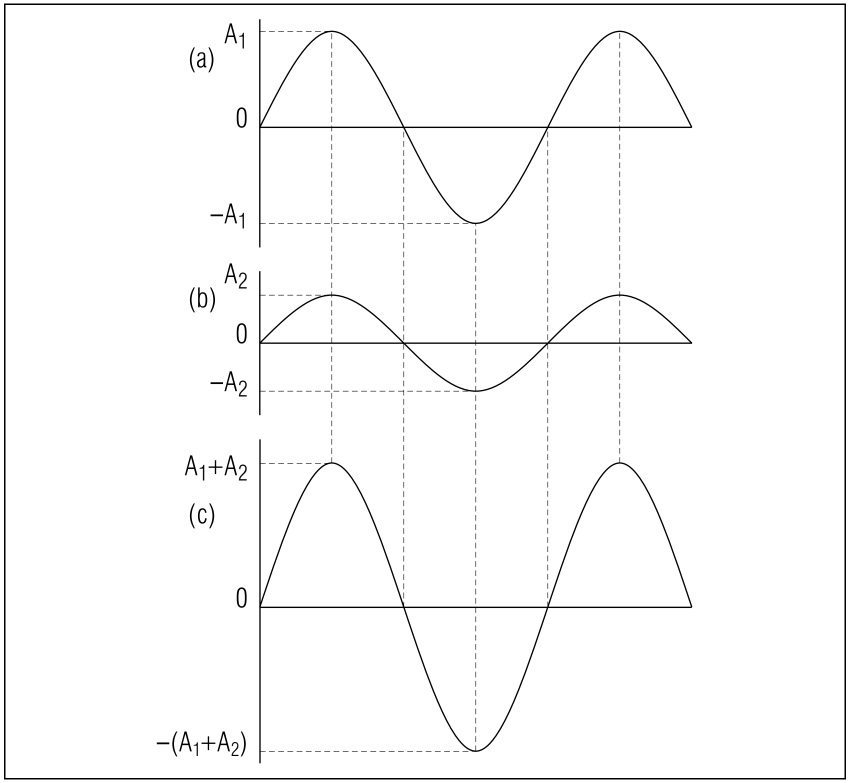


Fig. A-1: (a), (b) Two sinusoidal waves in phase. (c) Resultant wave equal to the sum of these waves.

If the waves have equal amplitudes, $A_1 + A_2 = 2A_1$. Then Eq. (A-3) becomes simply, for $A_1 = A_2$,

$$I_{\max} = 4\gamma A_1^2 = 4I_1, \qquad \text{(A-5)}$$

the result previously obtained in Relation (F-3) of Unit 430.

► *Waves one-half cycle out of phase*

Suppose that the two waves are one-half cycle "out of phase" (i.e., that their effective phase difference is one-half cycle) so that the maximum of one wave occurs always at the same instant as the minimum of the other wave. (See Fig. A-2a and Fig. A-2b.) Then the amplitude A of the resultant wave has its minimum possible value $A_{\min} = A_1 - A_2$. (See Fig. A-2c.) Correspondingly, the intensity I of the resultant wave has then also its minimum possible value:

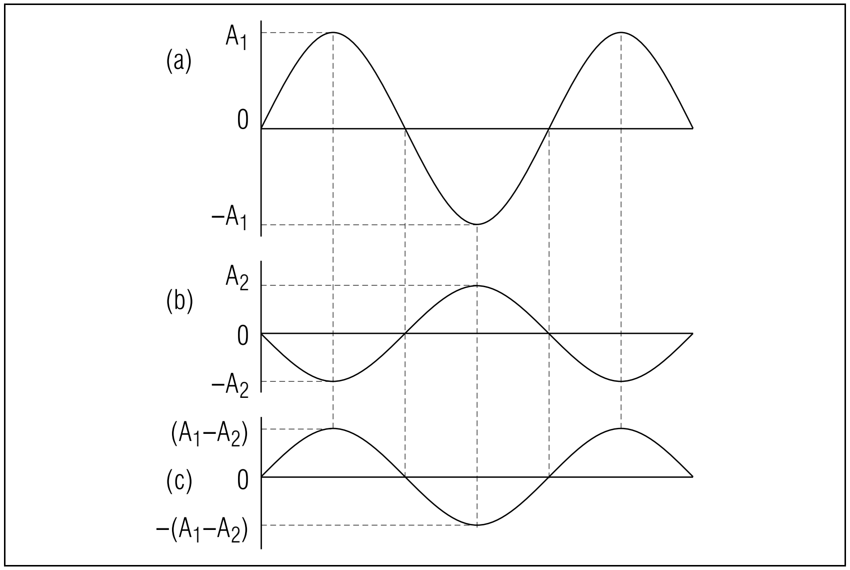


Fig. A-2: (a) and (b), Two sinusoidal waves one-half cycle out of phase; (c) Resultant wave equal to the sum of the waves in (a) and (b).

$$\boxed{I_{\rm min} = \gamma A_{\rm min}^2 = \gamma (A_1 - A_2)^2\,.} \qquad \text{(A-6)}$$

Thus

$$I_{\rm min} = \gamma (A_1^2 + A_2^2 - 2A_1A_2),$$

or

$$I_{\rm min} = I_1 + I_2 - 2\gamma A_1 A_2, \qquad \text{(A-7)}$$

where $I_1 = \gamma A_1^2$ and $I_2 = \gamma A_2^2$ are the intensities of the individual waves. This minimum possible resultant intensity is obviously *smaller* than the sum of the individual intensities [by the last term in Eq. (A-7)] and corresponds to the most extreme case of destructive interference.

If the waves have equal amplitudes, $A_1 - A_2 = 0$. Then Eq. (A-6) becomes simply, for $A_2 = A_1$,

$$I_{\rm min} = 0, \qquad \text{(A-8)}$$

the result previously obtained in text section F of Unit 430.

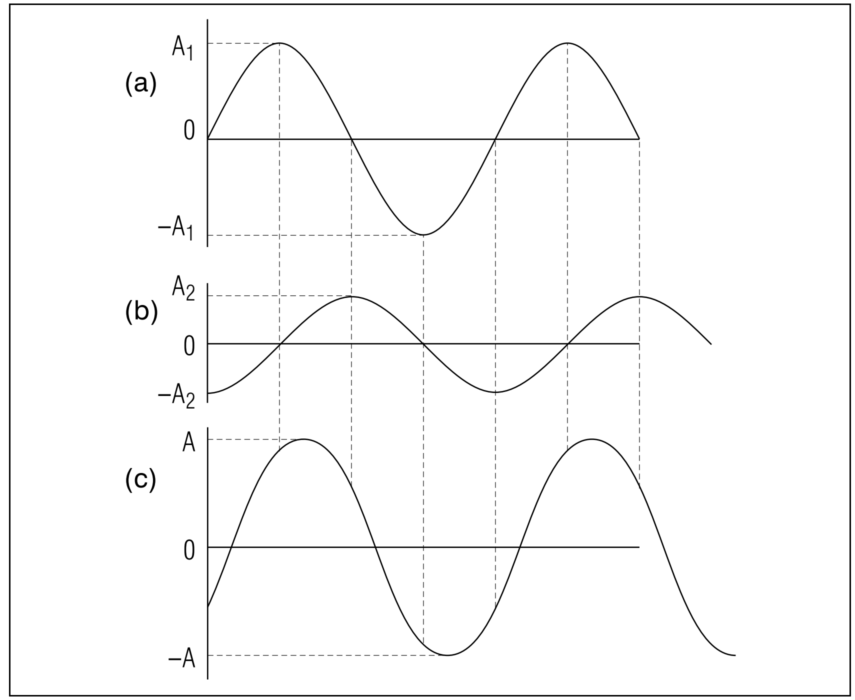


Fig. A-3: (a) and (b), Two sinusoidal waves one-fourth cycle out of phase; (c) Resultant wave equal to the sum of the waves in (a) and (b).

► *Waves one-fourth cycle out of phase*
As a final example, suppose that the two waves are one-fourth cycle out of phase so that the maximum of one wave occurs always when the other wave is zero. (See Fig. A-3a and Fig. A-3b.) Then the resulting wave (shown in Fig. A-3c) is sinusoidal with an amplitude A and an intensity I intermediate between the values corresponding to extreme constructive and destructive interference. [Indeed, one can show that $A^2 = A_1^2 + A_2^2$ so that $I = I_1 + I_2$. Thus there is no interference in this special case.]

NON-INTERFERENCE OF INCOHERENT WAVES

Consider now the situation where the two waves are *incoherent*. In the course of time the phase difference between the waves varies then in random fashion and is equally likely to assume any value. Correspondingly, the waves then tend to interfere as often constructively as destructively, with the net result that the interference effects cancel on the average. Hence

$$\boxed{\text{for incoherent waves, } I = I_1 + I_2\,.} \quad \text{(A-9)}$$

In other words, because of the cancellation of all interference effects, the resultant intensity of *incoherent* waves is simply equal to the sum of their individual *intensities*. (For example, the resultant intensity of light coming from two different lamps is simply equal to the sum of the intensities due to each lamp since the light from these lamps is incoherent.)

► *Discussion*

Let us examine the preceding conclusion more closely. Since two incoherent waves interfere as often constructively as destructively, one expects (and can verify in greater detail) that the resultant intensity of the waves is the average of the extreme values for constructive and destructive interference, i.e., that

$$I = \frac{1}{2}(I_{\rm max} + I_{\rm min}). \quad \text{(A-10)}$$

If we use the results Eq. (A-4) and Eq. (A-7) for $I_{\rm max}$ and $I_{\rm min}$, Eq. (A-10) yields indeed the result $I = I_1 + I_2$ since the last terms in Eq. (A-4) and Eq. (A-7) have opposite signs and thus cancel.

Resultant Amplitude and Intensity of a Wave

A-1 Two sinusoidal waves of the same frequency arrive at the same point, the amplitude A_2 of the second wave being twice as large as the amplitude A_1 of the first wave so that $A_2 = 2A_1$. (a) Express the intensity I_2 of the second wave in terms of the intensity I_1 of the first wave. (b) Suppose that the two waves are coherent and in phase (so that the amplitude and intensity of the resultant wave are maximum.) Express the amplitude $A_{\rm max}$ of this resultant wave in terms of A_1. Express the intensity $I_{\rm max}$ of this wave in terms of the intensity I_1 of the first wave. (c) Suppose that the two waves are coherent and one-half cycle out of phase (so that the amplitude and intensity of the resultant wave are minimum). Express the amplitude $A_{\rm min}$ of this resultant wave in terms of A_1. Express the intensity $I_{\rm min}$ of this wave in terms of I_1. (d) Suppose that the two waves are incoherent. Express the intensity $I_{\rm inc}$ of the resultant wave in terms of I_1. (e) What is the average value $(I_{\rm max}+I_{\rm min})/2$ of the intensities found in the two extreme cases of parts a and b? Compare this average value with the resultant intensity $I_{\rm inc}$ of the incoherent waves in part c. *(Answer: 4) (Suggestion: [s-3])*

A-2 Each of the two sinusoidal sound waves, of the same frequency, arriving at a given point P from two loudspeakers has an intensity of $10^{-4}\,{\rm watt}/m^2$. (a) What is the intensity of the resulting sound heard at P if the two sound waves at P are coherent and in phase? (b) If they are coherent and one-half cycle out of phase? (c) If they are incoherent? *(Answer: 2) (Suggestion: [s-1])*

A-3 (a) A loudspeaker, connected to an audio-amplifier, emits a sound wave corresponding to the tone of middle C and produces an intensity of $10^{-5}\,{\rm watt}/m^2$ at some point P. Suppose that a second identical loudspeaker is placed next to the first loudspeaker (at a distance much less than the wavelength) and is connected to the same amplifier in exactly the same way as the first loudspeaker (so that each speaker now produces an intensity of $10^{-5}\,{\rm watt}/m^2$ at the point P). What then is the intensity of the resulting sound heard at P? (b) Two violinists stand side-by-side, separated by a distance much less than the wavelength of the tone of middle C played by each of them. If the intensity of the sound produced by each of them at a point P is $10^{-5}\,{\rm watt}/m^2$, what is the intensity of the resulting sound heard at P? *(Answer: 7) (Suggestion: [s-5])*

More practice for this Capability: [p-1], [p-2], [p-3]

SECT.

B INTERFERENCE AND PATH DIFFERENCE

In the preceding section we studied the interference effects produced by two waves at some given point. We now examine the interference effects that waves from two sources produce at *various* points.

► *Small coherent sources*

Consider two sources, S_1 and S_2, each small compared to the wavelength of the waves they emit. They are a distance d apart. The waves they emit are coherent: the two sources' waves are of the same frequency (and thus of the same wavelength λ in the surrounding medium) and the two waves are emitted *in phase* That means, for example, that a wave maximum is always emitted at one source at the same instant that a wave maximum is emitted at the other source. The wave fronts of the spherical waves emitted by two such sources are illustrated in Fig. B-1.

► *Path difference*

We now focus attention at any point P that is a distance r_1 from source S_1 and a distance r_2 from source S_2 (see Fig. B-2a.) During the time a wave travels from a source to P, the phase of the wave changes by an amount dependent on the distance traveled by the wave (changing by one cycle while the wave travels a distance of one wavelength λ). If the distances r_1 and r_2 traversed by the two waves are different, the phases of the waves change by different amounts. Although the two waves leave the sources with the same phase, they arrive at P with a phase difference determined by the difference in the distances traveled by the waves, i.e., by their "path difference."

Def. **Path Difference**: $\Delta r = r_2 - r_1$ (B-1)

► *Intensity maxima*

If the path difference Δr of P from the two sources is equal to 0, or, $\pm\lambda$, $\pm 2\lambda$, ..., the corresponding phase difference is zero or some integral number of cycles. Hence the waves leaving the sources in phase arrive at P also in phase. Accordingly, they interfere there constructively to produce a maximum intensity $I_{\rm max}$. The points where the intensity is maximum are thus all those for which the path difference Δr satisfies this condition:

$$I = I_{\rm max} \quad \text{if} \quad \Delta r = 0,\ \pm\lambda,\ \pm 2\lambda,\ \ldots \tag{B-2}$$

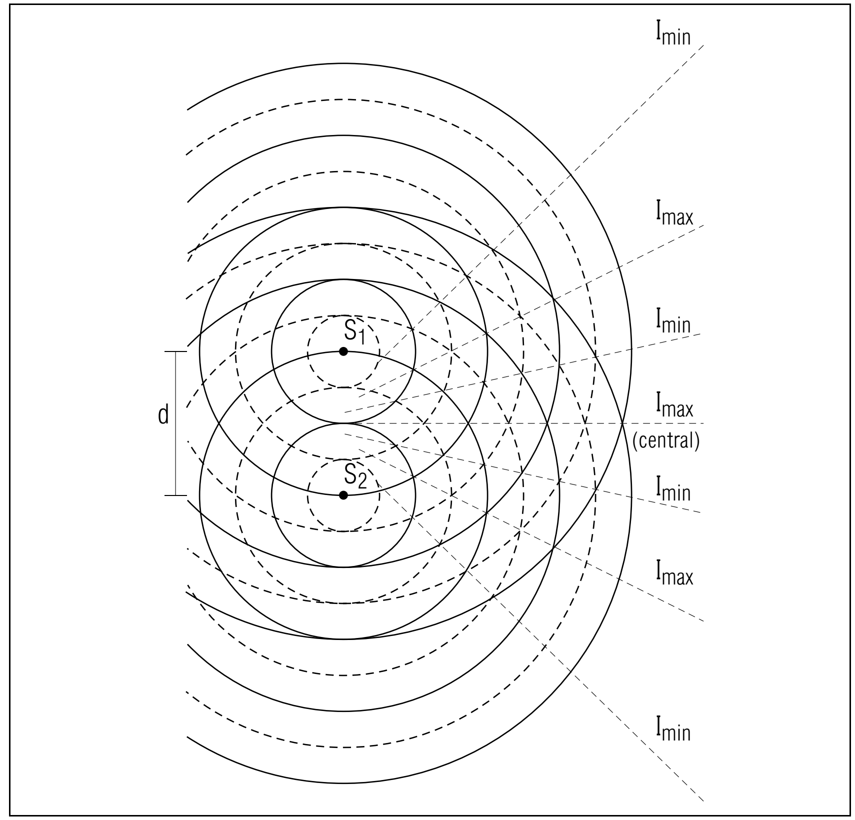


Fig. B-1: Wave fronts of waves emitted in phase from two sources.

These points are on the dotted lines marked $I_{\rm max}$ in Fig. B-1.

► *Intensity minima*

If the path difference Δr of P from the two sources is equal to $\pm(1/2)\lambda$, or $\pm(3/2)\lambda$, or $\pm(5/2)\lambda$,..., the corresponding phase difference produced in the waves is some odd number of half cycles, Hence the waves leaving the source in phase arrive at P so as to be half a cycle out of phase. Accordingly, they interfere there destructively so as to produce a minimum intensity $I_{\rm min}$. The points where the intensity is minimum are thus all those for which the path difference satisfies this condition:

$$I = I_{\rm min} \quad \text{if} \quad \Delta r = \pm\frac{1}{2}\lambda,\ \pm\frac{3}{2}\lambda,\ \pm\frac{5}{2}\lambda,\ \ldots \tag{B-3}$$

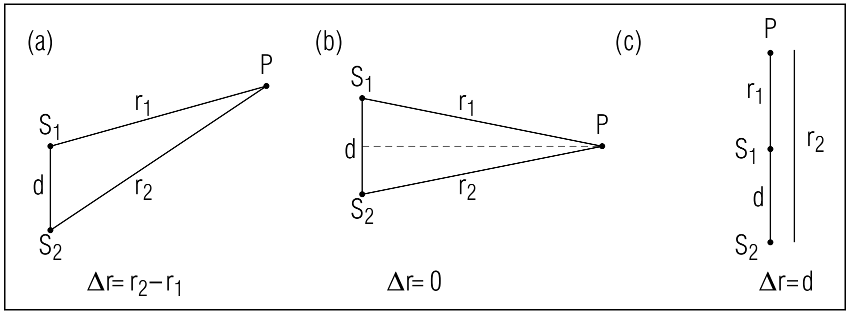


Fig. B-2: Path difference Δr of a point P from two sources. (a) Arbitrary point. (b) Point equidistant from the sources. (c) Point in line with the sources.

These points are those lying in Fig. B-1 on the dotted lines marked by $I_{\rm min}$.

► *Interference pattern*

The wave fronts in Fig. B-1 or in the photograph of Fig. B-3 illustrate pictorially how the waves from the two sources interfere at various points in space. For example, if the waves leave the sources with the same phase and travel the *same* distance, they must arrive in phase. Hence at all points equidistant from the sources (i.e., the points on the dotted "central" line in Fig. B-1) the waves interfere constructively to produce maximum intensity. On each side of this central line there is another line along which the waves interfere constructively because one wave travels *one* wavelength farther than the other. On each side of this central line there is also another line along which the waves interfere constructively because one wave travels *two* wavelengths farther than the other.

On the other hand, between these lines there are other lines along which the waves interfere destructively because one wave travels farther than the other one by $(1/2)\lambda$, $(3/2)\lambda$, ...

VALUE OF PATH DIFFERENCE

Let us examine more closely the path difference Δr between two sources and any point P, since Δr determines crucially the interference observed at P.

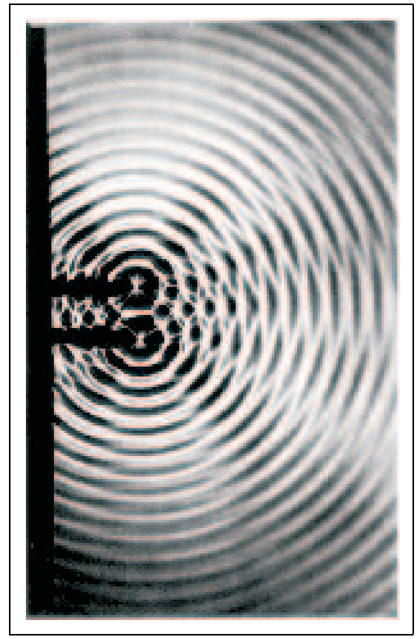


Fig. B-3: Photograph showing interference of waves on a water surface. The waves are produced by two sources (rods moving repetitively up and down through the water surface).

► *Possible values of* Δr

The *minimum* magnitude of the path difference is zero. As indicated in Fig. B-2b, this path difference occurs when the point P is equidistant from both sources, i.e., when the line from the midpoint between the sources to P is perpendicular to the line joining the sources. The *maximum* possible magnitude of the path difference is equal to the distance d between the sources. As indicated in Fig. B-2c, this path difference occurs when P is in line with the two sources. Thus the magnitude of Δr lies in the range

$$0 \leq |\ \Delta r\ | \leq d \qquad \text{(B-4)}$$

► Δr *for distant point*

The path difference $\Delta r = r_2 - r_1$ can be found particularly easily when the point P is far from the sources (i.e., so that the distance of P from either source is much larger than the separation d between the sources). As indicated in Fig. B-4, the lines from each source to P are then nearly parallel. Hence their common direction can be conveniently specified by the angle θ which these lines make with respect to a "central

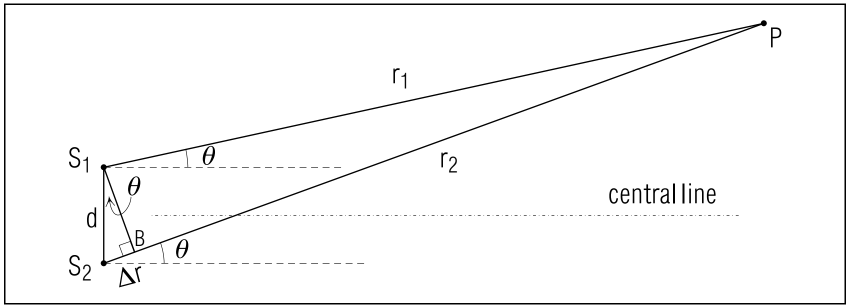


Fig. B-4: Path difference of a point P from two sources when the distance of P from these sources is much larger then their separation d.

line" perpendicular to the line joining the sources. (See Fig. B-4.)

How then is the path difference Δr related to the separation d between the sources and to the angle θ which specifies the direction of the point P relative to the sources?

► *Calculation of* Δr

To find an approximate value of the path difference Δr, we draw in Fig. B-4 the line S_1B perpendicular to the nearly parallel lines S_1P and S_2P. The distances S_1P and BP are then both nearly equal to r_1. Hence the distance $S_2B = r_2 - r_1 = \Delta\, r$ is the path difference we wish to find. In the right triangle S_1S_2B the hypotenuse S_1S_2 is equal to d. To find the angle S_2S_1B, labeled θ in Fig. B-4, we note that S_1B makes an angle of 90° with S_1P, while S_1S_2 makes an angle of $\theta + 90°$ with S_1P (since S_1B makes an angle θ with the central line which is perpendicular to S_1S_2). Hence the angle S_2S_1B, equal to the difference of these angles, is simply θ. Thus the sine of this angle in the right triangle S_2S_1B is

$$\sin\theta = \frac{\overline{S_2B}}{\overline{S_1S_2}} = \frac{\Delta r}{d}$$

so that

$$\boxed{\Delta r = d\sin\theta} \qquad \text{(B-5)}$$

This result is a good approximation for the path difference as long as the distance of the point P from the sources is much larger than their separation.

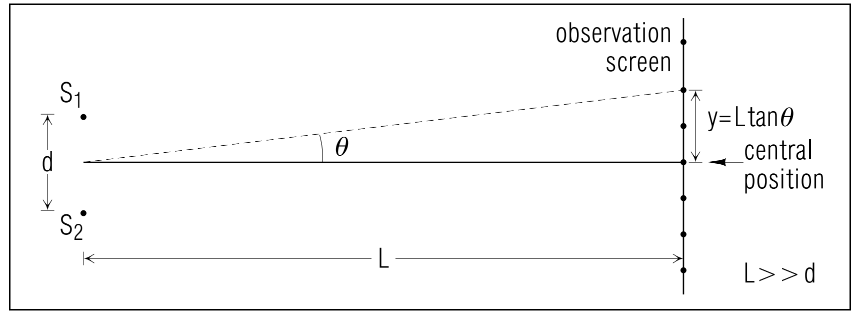


Fig. B-5: Positions of intensity maxima and minima along a line at a large distance from two sources.

► Δr *and angle* θ

The path difference Δr between the point P and the two sources depends thus in a simple way on the angle θ specifying the direction of P relative to the sources. In particular, if P is along the central line where $\theta = 0$, $\sin\theta = 0$ so that $\Delta r = 0$. But if P is in line with the sources, $\theta = \pm 90°$ so that $\sin\theta = \pm 1$ and $\Delta r = \pm d$. These conclusions agree with our previous comments, as illustrated in Fig. B-2b and Fig. B-2c.

LOCATION OF INTERFERENCE MAXIMA AND MINIMA

Since $\sin\theta = \Delta r/\mathrm{d}$, Eq. (B-2) and Eq. (B-3) allow us to find the angles of the lines along which the intensity produced by the waves from the sources is either maximum or minimum (e.g., of the dotted lines in Fig. B-1). Thus Eq. (B-2) and Eq. (B-3) imply that:

$$\begin{aligned} I &= I_{\max} \quad \text{if} \quad \sin\theta = 0,\ \pm\frac{\lambda}{d},\ \pm 2\frac{\lambda}{d},\ \ldots \\ I &= I_{\min} \quad \text{if} \quad \sin\theta = \pm\frac{1}{2}\frac{\lambda}{d},\ \pm\frac{3}{2}\frac{\lambda}{d},\ \pm\frac{5}{2}\frac{\lambda}{d},\ \ldots \end{aligned} \qquad \text{(B-6)}$$

► *Observation of interference*

The interference effects can be conveniently studied by observing the intensity produced along some line parallel to the sources at some large distance L from them. (See Fig. B-5.) As a result of interference, one then observes along the line a succession of intensity maxima and minima are specified by the angles in Eq. (B-5) or, equivalently, by the distances $y = L\tan\theta$ measured along the line from the central position corresponding to the angle $\theta = 0$.

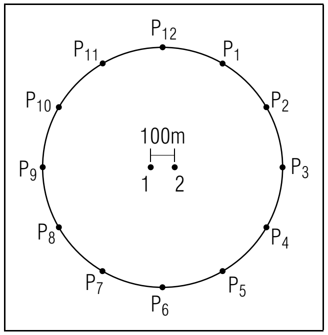


Fig. B-6.

Interference due to Two Sources (Cap. 2)

B-1 *Interference and path difference (sources in phase):* Two small loudspeakers emit sound waves which have a wavelength of 2 m and which are in phase. At each of the following points, is the intensity of the resultant wave a maximum as a result of constructive interference, or a minimum as a result of destructive interference? (a) At a point at a distance of 6 m from each of the speakers. (b) At a point at a distance of 4 m from the first speaker and 6 m from the second speaker. (c) At a point 4 m from the first speaker and 5 m from the second speaker. (d) At a point 6 m from the first speaker and 3 m from the second speaker. *(Answer: 3)*

B-2 *Interference and path difference (sources out of phase):* In the preceding problem B-1, Suppose that the speakers emit the same sound waves one-half cycle out of phase. What then would be the answers to the questions in that problem? *(Answer: 1) (Suggestion: [s-2])*

B-3 *Interference at various points around two sources:* Two radio antennas are separated by a distance of 100 m. Figure B-6 shows various points P_1, $\ldots P_{12}$ (numbered like the numbers on a clock), at a large distance of 3000 m from the antennas and equally spaced around a circle. The antennas emit radio waves which have a wavelength of 100 m. The amplitudes of these waves at all the points P are nearly the same. (a) What is the path difference from the antennas at the point P_{12} which is on the central line equidistant from the antennas? (b) What is the path difference at the point P_3? (c) What is the path difference at the point P_1 at an angle of 30° from the central line? (d) Does the intensity I observed at the point P_{12} correspond to a maximum or minimum possible intensity due to the two antennas? (e) What is the intensity at P_6? (f)

What is the intensity observed at P_3 and at P_9? (g) What is the intensity at P_1? (h) What is the intensity at P_{11}, at P_5, and at P_7? *(Answer: 5) (Suggestion: [s-9])*

B-4 *Dependence of Δr on d and θ:* (a) For a given separation between two sources, is the path difference from a point P to these sources larger or smaller if P is located at a larger angle θ from the central line? (b) For a given point at a large distance from two sources, is the path difference of P from these sources larger or smaller, if the separation between the sources is larger? (c) Suppose that the path difference of a point P from two sources is supposed to be $\lambda/2$ (so that the waves from the sources in phase interfere at P destructively). If P is at a larger angle θ from the central line, must the separation between the sources then be larger or smaller? *(Answer: 10)*

B-5 *Location of observed interference:* Two sources, emitting in phase waves of wavelength λ, are separated by a distance $d = 2\lambda$ (as illustrated in Fig. B-1). Points where the intensity is maximum are then located along the central line for which $\theta = 0$. (a) Intensity maxima are then also observed at distant points at which the magnitude of the path difference is λ and $2\,\lambda$. At what angles from the central line are these points located? (b) Intensity minima are observed at distant points where the magnitude of the path difference is $(1/2)\lambda$ and $(3/2)\lambda$. At what angles from the central line are these points located? *(Answer: 6)*

B-6 *Interference on a distant screen:* Consider in Fig. B-5 interference maxima and minima located at small angles θ (so small that $\tan\theta \approx \sin\theta$). (a) What then are the values of y specifying on the screen the positions of the central intensity maximum and the two neighboring intensity maxima? (b) What are the values of y specifying the positions of the two intensity minima next to the central maximum? Express your answers in terms of L, d, and λ. *(Answer: 11)*

SECT.

C INTERFERENCE DUE TO TWO SOURCES

By using the results of the preceding section, we can now discuss all the features of the interference of waves from two sources. (For simplicity, we assume throughout that the interference effects are observed at points whose distance from the sources is much larger than the separation d of the sources.)

SOURCES IN PHASE

► *Location of maxima and minima*

Let us first consider the case where the two sources emit the waves *in phase*. Then the interference maxima and minima are located at points whose positions relative to the sources are specified by the angles θ in Eqs. (B-6). Thus:

$$\begin{aligned} I = I_{\rm max} \quad &\text{if} \quad \sin\theta = 0,\ \pm\frac{\lambda}{d},\ \pm 2\frac{\lambda}{d},\ \ldots \\ I = I_{\rm min} \quad &\text{if} \quad \sin\theta = \pm\frac{1}{2}\frac{\lambda}{d},\ \pm\frac{3}{2}\frac{\lambda}{d},\ \pm\frac{5}{2}\frac{\lambda}{d},\ \ldots \end{aligned} \qquad \text{(C-1)}$$

Thus the intensity is maximum along the central line $\theta = 0$ (where all points are equidistant from the sources). There is a succession of other intensity maxima located symmetrically on both sides of this line. The intensity minima are located between these intensity maxima.

According to Eqs. (C-1), the separation between adjacent interference maxima (or adjacent interference minima) increases corresponding to the ratio λ/d of the wavelength λ compared to the separation d between the sources. If the separation d between the sources is kept fixed but the wavelength λ is made larger, the separation between the adjacent interference maxima is thus correspondingly also *larger*. On the other hand, if the wavelength λ is kept fixed but the separation d between the sources is made larger, the separation between the adjacent interference maxima is correspondingly *smaller*.

► *Case where $d < \lambda/2$*

The angle $\theta_{\rm min}$ between the central interference maximum at $\theta =$ 0 and the first interference minimum is such that

$$\sin\theta_{\rm min} = \frac{1}{2}\left(\frac{\lambda}{d}\right) \qquad \text{(C-2)}$$

This angle $\theta_{\rm min}$ is larger if the ratio λ/d is larger. Indeed, if $(1/2)(\lambda/d) > 1$, $\sin\theta_{\rm min}$ exceeds its maximum permissible value of 1. This means that *no* intensity minimum is then observed anywhere. Indeed, it is easy to see why this happens. In order to observe destructive interference anywhere, the path difference Δr from the sources must somewhere be at least as large as $(1/2)\lambda$. But the maximum possible path difference is equal to d. Hence destructive interference can be observed anywhere only if d is at least as large as $(1/2)\lambda$. If d is much smaller than $(1/2)\lambda$ the sources are so close together that the path difference between them and any point is always negligible compared to λ. Then the waves, leaving the sources in phase, arrive everywhere nearly in phase and thus never interfere destructively.

► *Intensities of maxima and minima*

Suppose that we observe the waves arriving in a small region far from the sources. Since the wave from each source travels then nearly the same distance, the amplitude of each wave decreases by nearly the same fraction. Hence the ratio of the amplitudes of the waves arriving in this region is nearly the same as the ratio of the amplitudes of the waves leaving the sources.

Suppose that the amplitudes A_1 and A_2 of the waves arriving in the region are the same (because the waves have equal amplitudes when leaving the sources). Then the observed intensity maxima and minima are, by Eq. (A-5) and Eq. (A-8), equal to

$$I_{\rm max} = \gamma(2A_1)^2 = 4I_1\ ,\ I_{\rm min} = 0 \qquad \text{(C-3)}$$

where I_1 is the intensity which would be observed from one wave alone.

If the waves arriving in the region have different amplitudes A_1 and A_2, the observed intensity maxima and minima are, by Eq. (A-3) and Eq. (A-6), equal to

$$I_{\rm max} = \gamma(A_1 + A_2)^2\ ,\ I_{\rm min} = \gamma(A_1 - A_2)^2 \qquad \text{(C-4)}$$

► *Energy conservation*

At points where the waves interfere constructively, the intensity $I_{\max}$ is larger than the sum $I_1 + I_2$ of the intensities produced by the individual sources. At points where the waves interfere destructively, the intensity $I_{\min}$ is smaller than the sum $I_1 + I_2$ of the intensities produced by the individual sources. But the average intensity $\bar{I}$, averaged over all points of space, is equal to

$$\bar{I} = \frac{1}{2}(I_{\max} + I_{\min}) = I_1 + I_2 \qquad \text{(C-5)}$$

[For example, if the waves have equal amplitudes, $\bar{I} = (1/2)(4I_1 + 0) = 2I_1$.]*

* Similarly, for waves with different amplitudes, the result Eq. (C-5) follows from Eq. (A-4) and Eq. (A-7).

Hence the conservation of energy is properly satisfied, despite the interference effects. Thus the two sources do emit a total power equal to the sum of the powers emitted by each. But this power does not travel from the sources uniformly in all directions. Instead, the emitted power is larger than the sum of the powers along those directions where the waves interfere constructively, and is smaller than the sum of the powers along those directions where the waves interfere destructively.

SOURCES ONE-HALF CYCLE OUT OF PHASE

Suppose that the waves leaving the sources are not in phase, but 1/2 cycle out of phase. What then would happen to the interface pattern? The two waves still acquire the previously discussed phase difference as a result of the difference Δr in the paths traveled by the waves. However, the waves have now also the additional phase difference of 1/2 cycle which they had when leaving the source. The result is then merely that the waves now interfere constructively at those points where they interfered destructively before, and vice versa. Thus the interference pattern is exactly the same as in the previous case of waves leaving the sources in phase, except that all previous intensity maxima are now intensity minima, and vice versa.

INFORMATION OBTAINABLE FROM AN INTERFERENCE PATTERN

The observation of an interference pattern allows one to deduce much information about the sources emitting the waves:

(1) As is apparent from Eqs. (C-1), the observation of the locations (specified by the angle θ) of the observed interference maxima and minima allows one to obtain information about the separation d between the sources.

(2) As is apparent from Eq. (C-4), the observation of the relative intensities of the intensity maxima and minima allows one to obtain information about the relative amplitudes of the wave emitted by the sources.

These comments indicate why (as discussed in the next unit) observations of the interference produced by X-rays emitted from molecules allow one to obtain important information about the arrangement of atoms in molecules of chemical or biological interest.

Interference due to Two Sources (Cap. 2)

C-1 *Dependence of interference pattern on λ and d:* Two sources, separated by a distance d, emit in phase coherent waves having a wavelength λ. (a) What is the magnitude of the smallest angle $\theta_{\min}$, measured from the central direction, along which the waves interfere destructively to yield an intensity minimum $I_{\min}$ of the resultant wave? (b) Suppose that the wavelength of the waves emitted by the sources were larger. Would the angle $\theta_{\min}$ then be larger or smaller? (c) For what wavelength λ_1 would the angle $\theta_{\min}$ be 90°? (d) Where would the interference minimum $I_{\min}$ be observed if the wavelength is larger than λ_1? (e) Suppose that the original wavelength (or frequency) of the emitted waves is kept constant, but that the separation between the sources is made larger. Would the angle $\theta_{\min}$ then be larger or smaller? *(Answer: 9)*

C-2 *Interference from two speakers:* Two small loudspeakers, connected to the two channels of a stereo amplifier, are separated by a distance of 3.0 m and emit in phase sound waves having a frequency of 400 Hz and the same amplitude. The waves then travel through the air with a speed of 340 m/s. At a distant point P_0, equidistant from both speakers, the observed intensity of the resultant wave is then maximum. (a) At what smallest-magnitude angle, measured from the line joining P_0 to the mid-point between the sources, is the intensity of the resultant wave minimum? (b) At what smallest-magnitude angle is the intensity of the resultant wave then again maximum? (c) Suppose that the frequency of the waves emitted by the speakers was twice as large (i.e., 800 Hz). What then would be the answer to part (b)? (d) Suppose that the speakers emit waves with the original frequency of 400 Hz, but that the amplitude of the wave emitted by one speaker was twice as large as the unchanged amplitude of the wave emitted by the other speaker. Would the magnitudes of the angles found in parts *a* and *b* then be larger, smaller or the same? (e) Would the previously observed intensity maxima then be larger, smaller, or the same? *(Answer: 13) (Suggestion: [s-7])*

C-3 *Frequency measurement by interference:* Two small loudspeakers, separated by a distance of 4.0 m, are connected to the same amplifier so as to emit two sinusoidal sound waves in phase. Because of interference, one then observes a maximum intensity along the central line (everywhere equidistant from the speakers) and again at an angle of 20° from this central line. Use this information, and the fact that the speed of sound in air is 340 m/s, to find the frequency of the waves emitted by the loudspeakers. *(Answer: 8)*

C-4 *Directional antennas:* Fig. C-1 shows two radio antennas, emitting radio waves with a wavelength λ, separated by a distance $1/2\,\lambda$. The four points P_1, ..., P_4 are far from the antennas and each antenna separately produces at each of these points a radio wave of intensity I. The following questions illustrate how the interference of the waves radiated by both antennas can be used to broadcast the radio signal preferentially along certain directions.

(a) Suppose that the antennas emit the radio waves in phase. What then is the intensity of the resultant radio wave at the points P_1 or P_3? What is the intensity at the points P_2 or P_4? (b) Suppose now that the antennas emit the radio waves 1/2 cycle out of phase. What then is the intensity of the resultant radio wave at the points P_1 or P_3? What is the intensity at the points P_2 or P_4? *(Answer: 14) (Suggestion: [s-6])*

More practice for this Capability: [p-4]

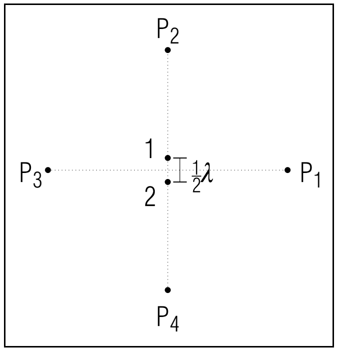


Fig. C-1.

SECT.

D INTERFERENCE DUE TO WAVES THROUGH HOLES

► *Hole Equivalent to Source*

A screen of some material is "opaque" to a wave if the wave cannot pass through the screen, but is absorbed or reflected. (For example, a screen opaque to a sound wave might be a sheet of rock-wool, or a screen opaque to a light wave might be a sheet of cardboard.) Suppose that a sinusoidal wave is incident upon such an opaque screen which contains a hole small compared to the wavelength. The wave then produces in the small hole a sinusoidal disturbance, which then produces in its neighborhood a similar disturbance, which then produces in its neighborhood a similar disturbance, ... Thus a spherical wave emanates from the small hole, as illustrated in Fig. D-1. Hence the small hole acts like a small source emitting a spherical wave in the region behind the screen.

► *Interference from two holes*

Suppose now that the opaque screen contains *two* small holes a distance d apart. These two holes are then equivalent to two small sources. Hence the waves emanating from these holes produce behind the screen exactly the same interference effects as the waves from the two sources discussed in the preceding sections.

► *Small Slits*

Suppose that the two holes in the screen are long *slits* (perpendicular to the plane of the paper in Fig. D-1) of width small compared to the wavelength. Then the situation is similar to that for small holes, except that all effects occur along lines perpendicular to the paper. Thus each slit acts like a linear source (emitting cylindrical waves) and interference maxima and minima occur along lines perpendicular to the paper.

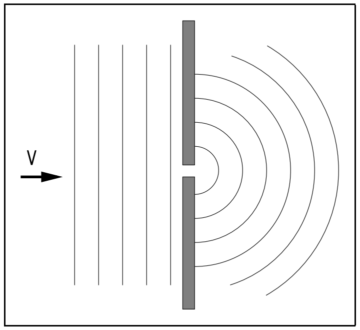


Fig. D-1: Wave emanating from a small hole in an opaque screen when a wave is incident upon this screen.

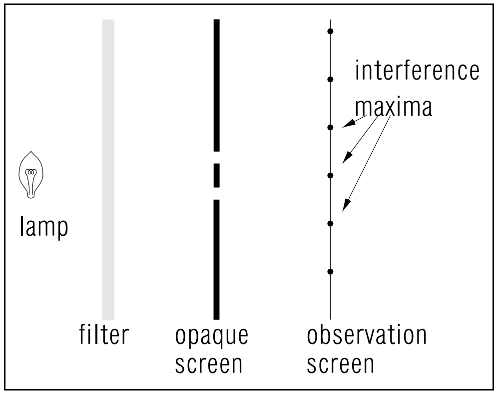


Fig. D-2: Experimental arrangement for observing the interference of light passing through two small holes.

INTERFERENCE OF LIGHT

► *Holes as coherent sources*

One can only observe the interference of light if the light comes from two *coherent* sources. Thus one cannot use two separate lamps since the light from them is incoherent. On the other hand, suppose that one uses a *single* lamp at a position equidistant from two small holes (or slits) in an opaque screen, as indicated in Fig. D-2. Then light emitted by any individual atom in the lamp arrives at both holes with the same phase. As a result, the holes act as coherent sources of light and the light waves emanating from them can produce interference.*

> * Light originally emitted by any individual atom can produce interference effects. Since the light emitted by *different* atoms is incoherent, the total intensity due to all these atoms is then the sum of their individual intensities. But this sum merely results in an increased intensity at every point, but does *not* affect the ratio of the intensities at two points, (i.e., does *not* destroy the interference effects due to light from any individual atom).

► *Interference experiment*

A lamp usually emits white light consisting of a mixture of different frequencies. To perform the interference experiment with light of one selected frequency (or corresponding wavelength), one needs only let

Fig. D-3: Photograph of interference maxima and minima produced by light passing through two slits.

the light from the lamp pass through a "filter" consisting of a colored glass plate which absorbs light of all frequencies except the frequency corresponding to the color of the plate. With such a filter, the holes in Fig. D-2 are then equivalent to two small sources emitting light of one particular wavelength. The light emanating from these holes then produces on some distant screen (e.g., on a photographic plate) a series of intensity maxima and minima caused by the interference of the light. (See Fig. D-3).

The preceding experiment demonstrating the interference produced by light was first performed in 1803 by Thomas Young (1773-1829), a remarkable English physician who made several important contributions to physics and also helped to decipher the Egyptian hieroglyphic language. Young's experiment was important for these reasons:

► *Wave properties of light*

(1) The experimental observation of alternating light intensity maxima and minima demonstrates conclusively that light has the wave properties responsible for interference. Thus the experiment rules out the possibility that light consists of particles traveling along definite paths.

For in that case the intensity observed at any point for light coming from two holes could never be less than the intensity for light coming from a single hole. (In the case of waves this *can* happen because light coming from the second hole can interfere destructively with light coming from the first hole.)

► *Measurement of* λ

(2) By knowing the distance d between the holes and measuring the separation between successive intensity maxima in the observed interference pattern, one can find the wavelength of light. Thus one finds that $\lambda \approx 7 \times 10^{-7}$ m for red light and $\lambda \approx 4 \times 10^{-7}$ m for violet light. From the known speed of light in vacuum, one can then calculate the corresponding frequency of light.*

> * Indeed, until very recently it has not been possible to measure directly the frequency of light since this frequency is so large. Hence the frequency of light has traditionally always been inferred from measurements from the wavelength of light.

Interference due to Two Sources (Cap. 2)

D-1 *Interference due to different colors:* Consider the experiment of Fig. D-2, illustrating the interference of light passing through two slits. Is the spacing between the adjacent intensity maxima, produced on the observation screen, larger if the experiment is performed with red light or with blue light? *(Answer: 12) (Suggestion: [s-8])*

D-2 *Measuring wavelength and frequency of light:* Light from a laser is incident perpendicularly on a blackened glass slide containing two very narrow identical transparent slits (formed by scratching away the black paint from the slide). The separation between the slits is 0.40 mm. The interference pattern observed on a screen located at a distance of 2.00 m behind the glass slide then shows alternating intensity maxima separated by a distance of 3.1 mm. (a) On the basis of this information, what is the wavelength of the light emitted by the laser? Express your answer in the unit "ångstrom" (where 1 ångstrom = 1 Å = 10^{-10} m). (b) What is the frequency of this light? *(Answer: 15) (Suggestion: [s-12])*

More practice for this Capability: [p-5], [p-6]

SECT.

E INTERFERENCE DUE TO REGULARLY SPACED SOURCES

► *N sources along a line*

Interference effects can be particularly pronounced and practically useful when one deals with waves coming from *many* sources. Let us then consider the situation of any number N of sources, each of them small compared to the wavelength, regularly spaced along a line so that the separation between any two adjacent sources is equal to d. (See Fig. E-1.) These sources emit *in phase* coherent waves of the same amplitude A_1 and of the same wavelength λ. We assume that the intensity resulting from these waves is observed at points P so far from the sources that lines drawn from the sources to P are nearly parallel. As indicated in Fig. E-1., the position of P relative to the sources can then be specified by the angle θ which these parallel lines make with the "central" line perpendicular to the line of the sources. What then are the interference effects observed at points P corresponding to various angles θ?

► *Constructive Interference*

Since all sources are separated by the same distance d, the path difference Δr between a distant point P and *any* pair of adjacent sources is the same, by Eq. (B-5) equal to $\Delta r = d\sin\theta$. Suppose then that this path difference is equal to some integral number of wavelengths, i.e., that $d\sin\theta = n\lambda$ where $n = 0, \pm1, \pm2, \ldots$. Then the waves emitted in phase by two adjacent sources arrive at P also in phase. Since this happens

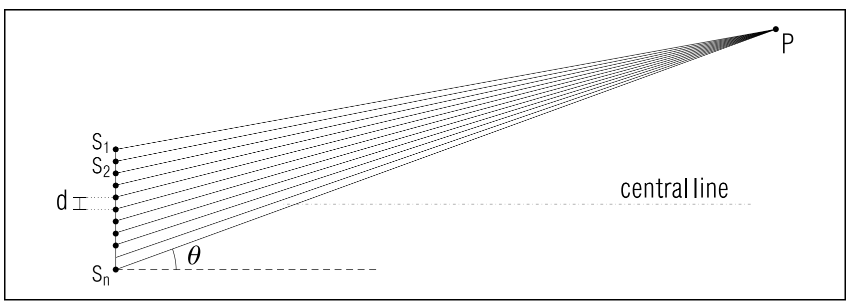


Fig. E-1: N equidistant sources and a point P at a large distance from these sources. (The point P is supposed to be so far that the lines from the sources to P are nearly parallel.)

for *any* pair of adjacent sources, *all* the waves emitted in phase by all N sources arrive at P in phase and thus interfere there constructively. By the superposition principle, the amplitude of the resultant wave has then its maximum possible value $A_{\rm max}$ equal to the sum of the N equal amplitudes A_1 of the individual waves from the separate sources. Thus $A_{\rm max} = NA_1$ and the corresponding intensity $I_{\rm min}$ of the resulting wave is

$$I_{\rm max} = \gamma A_{\rm max}^2 = \gamma(NA_1)^2 = N^2(\gamma A_1^2) = N^2 I_1 \qquad \text{(E-1)}$$

where $I_1 = \gamma A_1^2$ is the intensity which would be observed at P from a single source alone.

► *Location of Interference*

To summarize, one observes complete constructive interference of the waves from all sources at the special angles θ:

$$\boxed{I = I_{\rm max} = N^2 I_1, \qquad \text{if} \qquad d\sin\theta = n\lambda} \qquad \text{(E-2)}$$

where $n = 0, \pm1, \pm2, \pm3, \ldots$

(The integer n, specifying the number of wavelengths of path difference for waves from adjacent sources, is called the "order" of the interference.) If $n = 0$, one thus observes always (irrespective of the magnitude of λ) maximum intensity along the central line corresponding to $\theta = 0$.*

* The reason is, of course, that any distant point P along this central line is equidistant from all the sources.

Symmetrically on both sides of this central maximum one then observes a series of similar intensity maxima corresponding to $n = \pm1, \pm2, \ldots$, i.e., corresponding to angles θ such that $d\sin\theta = \pm\lambda, \pm2\lambda, \ldots$.

► *Intensity of Interference*

By Eq. (E-1), the intensity $I_{\rm max}$ of the resultant wave is N^2 times larger than the intensity of a wave from a single source. [This is because the intensity is proportional to the *square* of the amplitude, which is N times larger than that due to a single wave.] If N is large, the intensity $I_{\rm max}$ produced by constructive interference is then *very much* larger than I_1. For example, if there are $N = 10^3$ sources, the intensity $I_textmax$ is 10^6 times larger than the intensity due to a single source alone!

► *Sharpness of Interference*

The directions specified by the angles in Eq. (E-2) are very special since the waves from all N sources arrive there precisely in phase. Thus all waves have there at any instant the *same* sign and the magnitude of their sum is correspondingly large. At angles slightly different from those in Eq. (E-2), the waves are no longer precisely in phase. Thus some of the waves are positive while others are negative, and the sum of these waves is markedly smaller. At angles halfway between those in Eq. (E-2), about as many waves are positive as negative, and the sum of these waves is markedly smaller. At angles halfway between those in Eq. (E-2), about as many waves are positive as negative so that the cancellation of these waves makes their sum nearly zero. If the number N of waves is large, the resulting intensity $I_{\max} = N^2 I_1$ at the special angles specified by Eq. (E-2) is thus very much larger than the intensity at appreciably different angles. Hence the interference effects are then very sharply defined locations.

DIFFRACTION GRATING

An opaque screen with N equally spaced slits in it is called a "diffraction grating."*

> * The word "diffraction" refers to the spreading of waves in various directions when such waves are incident upon some object.

Suppose that a lamp is placed far from the grating so as to be nearly equidistant from all its slits, i.e., so that the wave fronts of light from the lamp are nearly parallel to the grating. (See Fig. E-1). Then light from any atom in the lamp arrives at any slit in phase. Thus the light emanating from the slits is the same as that produced by N coherent sources in phase. Accordingly, one then observes sharp interference maxima at the angles specified by Eq. (E-2).

► *Practical grating for light*

A practical grating for light is made by ruling on a transparent glass plate a series of equidistant parallel scratches. The remaining transparent regions between the scratched lines constitute then the slits of the grating. (For example, if 4000 lines per centimeter are ruled on a grating, the distance d between adjacent slits is 1/4000 centimeter.)*

> * Although the slits between the scratches may not be small compared to the wavelength of light, the main characteristics of the interference pattern remain unaffected.

Such a diffraction grating is useful for many practical applications:

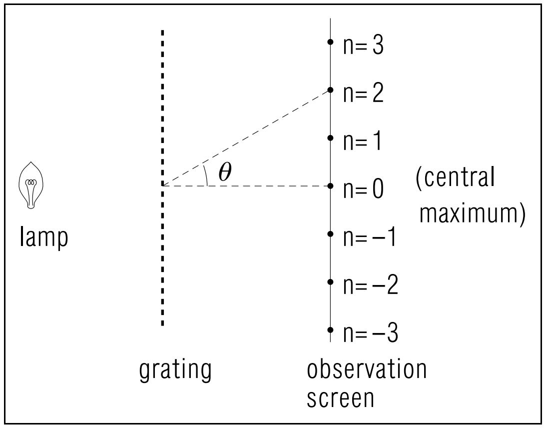


Fig. E-2: Interference maxima produced on an observation screen by light passing through a diffraction grating.

► *Precise measurement of λ*

By measuring the angles θ at which the interference maxima are observed, one can use the known grating spacing d to determine the wavelength λ of the light passing through the grating. If the number N of slits is large the interference maxima are located at sharply defined positions. Hence the measurement of the wavelength can be very precise.

► *Grating spectrometer*

Suppose that light, consisting of a mixture of waves of different wavelengths, is incident on a diffraction grating. Then all such waves interfere constructively along the central direction $\theta = 0$, corresponding to interference for $n = 0$ in Eq. (E-2). But for the various higher orders $n = 1, 2, \ldots$, the angle θ at which constructive interference occurs depends on the wavelength λ being larger for light with larger wavelength than for light with smaller wavelength. If light consisting of a mixture of wavelengths is incident on the grating, the light emerging from the grating is then deflected along different directions corresponding to different wavelengths. [A device which uses a diffraction grating to separate and measure the component wavelengths (or frequencies) of light is called a "grating spectroscope" or "grating spectrometer."] For example, if white light (which consists of a uniform mixture of light of all visible wavelengths) is incident on a diffraction grating, the light emerging from the grating in every order $n = 1, 2, \ldots$ is spread out in the colors of the rainbow, with violet light (which has the smaller wavelength) appearing at an angle θ smaller than red light (which has the larger wavelength).

► *Chemical analysis*

Different atoms or molecules emit light having distinct frequencies (and corresponding wavelengths) characteristic of the particular kind of atom or molecule. Similarly, different atoms or molecules also absorb light having distinct frequencies (and corresponding wavelengths) characteristic of the particular kind of atom or molecule. Hence it is possible to identify various kinds of atoms or molecules by using a grating spectrometer to analyze the wavelengths of light *emitted* by such atoms or molecules (i.e., by doing "emission spectroscopy"). Similarly, it is possible to identify particular atoms or molecules by analyzing the wavelengths of light *absorbed* by such atoms or molecules (i.e., by doing "absorption spectroscopy").

By thus using a grating spectroscope to analyze light which has interacted with atoms or molecules, one obtains a method of chemical analysis extremely useful for identifying the kinds of atoms or molecules present in various materials. This spectroscopic method is rapid, accurate, nondestructive, and needs only small amounts of material. Furthermore, the method can be used for the analysis of inaccessible systems. For example, one can identify the kinds and relative amounts of atoms present in a distant star by analyzing the light arriving from this star at the surface of the earth.

Interference due to Regularly Spaced Sources (Cap. 3)

E-1 *Spectrum produced by a grating:* Visible white light consists of a mixture of light with wavelengths ranging from about 4000 Å (corresponding to violet light) to about 7000 Å (corresponding to red light), where $1\,\text{Å} = 10^{-10}\,\text{m}$. Suppose that such white light is incident perpendicularly upon a diffraction grating having 4000 lines per centimeter. (a) For interferences produced in first order, at what angle (from the direction of the incident light) does the violet light emerge? At what angle does the red light emerge? Over what angular range is this first-order spectrum thus spread out? (b) Answer the same questions for the interference produced in second order. (c) Answer the same questions for the interference produced in the third order. (d) Is there any overlap between the spectrum produced in first order and that in second order? (e) Is there any overlap between the spectrum produced in second order and that in third order? *(Answer: 19) (Suggestion: [s-14])*

E-2 *Measurement of wavelength:* White light passes through a substance and is then incident perpendicularly on a diffraction grating having 5000 lines per centimeter. The first-order spectrum of light emerging from the grating then shows all the colors of the rainbow spread out at various angles, except for the fact that darkness is observed at the particular angle of 17°. Hence light of some particular wavelength λ must be missing from the white light incident on the grating because this light was absorbed by passing through the substance. On the basis of this information, what is the wavelength λ of light absorbed by the molecules in the substance? *(Answer: 17)*

E-3 *Design of a grating:* It is desired to design a diffraction grating so that yellow light, having a wavelength of $6.00 \times 10^{-7}\,\text{m}$ produces in second order an interference maximum at an angle of 30.0° from the direction of the original light incident perpendicularly on the grating. (a) What must be the spacing between the adjacent slits of such a grating? (b) If the grating is to have a width of 1 inch (i.e., 2.54 cm), what should be the approximate number of parallel lines which must be scratched on a glass plate of this width? (c) If green light is incident on this grating, would the resulting interference maximum produced in second order be at an angle equal to, larger than, or smaller than 30.0°? (d) suppose that one used instead a grating with the *same* separation between slits, but only 1/2 inch wide. If this grating were illuminated with yellow light in exactly the same way, would the second-order interference maximum occur at an angle equal to, larger than, or smaller than 30.0°? Express the intensity I of this interference maximum in terms of the intensity I_0 observed with the original 1 inch grating. *(Answer: 22) (Suggestion: [s-16])*

More practice for this Capability: [p-7], [p-8], [p-9]

SECT.

F SUMMARY

DEFINITIONS

coherent waves; Def. (A-1)

incoherent waves; Def. (A-2)

path difference; Def. (B-1)

IMPORTANT RESULTS

Resultant intensity of two coherent waves: Eq. (A-3), Eq. (A-6)

in phase $I_{\rm max} = \gamma(A_1 + A_2)^2$

1/2 cycle out of phase: $I_{\rm min} = \gamma(A_1 - A_2)^2$

Resultant intensity of two incoherent waves: Eq. (A-9)

$I = I_1 + I_2$ (no interference)

Interference of two coherent sources in phase: Eq. (B-2), Eq. (B-3)

$I = I_{\rm max}$ if $\Delta r = 0, \pm\lambda, \pm 2\lambda, \ldots$

$I = I_{\rm min}$ if $\Delta r = \pm\frac{1}{2}\lambda, \pm\frac{3}{2}\lambda, \pm\frac{5}{2}\lambda, \ldots$

Path difference for point far from two sources:

$\Delta r = r_2 - r_1 = d\sin\theta$ (if $r_1, r_2 \gg d$)

Small hole is equivalent to small source. (Sec. D)

Interference far from N coherent sources in phase: Eq. (E-2)

$I = I_{\rm max} = N^2 I_1$ if $d\sin\theta = n\lambda$ where $n = 0, \pm 1, \pm 2, \ldots$

USEFUL KNOWLEDGE

Two-slit interference of light. (Sec. D)

Diffraction grating and its applications. (Sec. E)

NEW CAPABILITIES

(1) Relate the amplitudes and intensities of two or more waves to the amplitude and intensity of the resultant waves when the individual waves are either coherent (in phase or 1/2 cycle out of phase) or incoherent. (Sec. A; [p-1], [p-2], [p-3])

(2) Consider two sources (or equivalent holes) emitting waves in phase or 1/2 cycle out of phase. Relate the locations of the interference maxima or minima to the path difference, and thus also to the distance between the sources and to the wavelength (or frequency). (Sects. B, C, and D, [p-4], [p-5], [p-6])

(3) Consider any number N of sources (or equivalent holes) regularly spaced along a line and emitting waves in phase. (a) Relate the locations of the interference maxima to the separation between the sources and to the wavelength (or frequency). (b) Relate the intensity of such an interference maximum to the intensity from a single source. (Sec. E, [p-7], [p-8], [p-9])

F-1 *Dependence of resultant intensity on individual intensities (Cap. 1):* (a) The waves from two coherent sources interfere constructively at a point P so that the resulting intensity at P is maximum. Suppose that the intensity of the waves emitted by one of the two sources is made larger. Does the intensity maximum at the point P then become larger, smaller, or remain the same? Explain. (b) The waves from these two coherent sources interfere destructively at some other point P' so that the resulting intensity at P' is minimum. Suppose that the intensity of the waves emitted by one of the two sources is made larger. Does the intensity at P' then become larger, smaller, or can either happen? Explain and give specific examples to illustrate your answer. *(Answer: 18) (Suggestion: [s-13])*

F-2 *Interference and wave speed:* Red light corresponding to a particular frequency is incident perpendicularly upon a diffraction grating and thus produces in first order an interference maximum at some angle θ_1 from the direction of the incident light. Suppose that the experiment is now performed with red light of the same frequency, but with the whole apparatus immersed in water in which the speed of light is smaller than in vacuum. Is the angle at which the first-order interference maximum occurs then larger than, smaller than, or equal to the original angle θ_1? *(Answer: 16) (Suggestion: [s-11])*

SECT.

G PROBLEMS

G-1 *Relation between resultant intensity and individual intensities:* Two coherent waves at the same point have respective intensities I_1 and I_2. (a) What is the maximum possible intensity of the resultant wave, i.e., the intensity resulting when the waves are in phase? (b) What is the minimum possible intensity of the resultant wave, i.e., the intensity resulting when the waves are 1/2 cycle out of phase? Express your answer in terms of the individual intensities I_1 and I_2. *(Answer: 24) (Suggestion: [s-18])*

G-2 *Amplitudes of waves determined from observed interference:* When two waves are in phase so as to interfere constructively, the observed intensity of the resultant wave is I_{max}. When these waves are 1/2 cycle out of phase so as to interfere destructively, the observed intensity of the resultant wave is I_{min}. (a) What then is the ratio A_1/A_2 of the wave with the smaller amplitude A_1 compared to the wave with the larger amplitude A_2? (b) What is the intensity I_1 of the wave with the smaller intensity? (c) What is the intensity I_2 of the wave with the larger intensity? (Express all your answers in terms of the observed intensities $I_{\rm max}$ and $I_{\rm min}$.) *(Answer: 21) (Suggestion: [s-20])*

G-3 *Acoustic Interferometer:* Figure G-1 shows an acoustic "interferometer," a device useful for measuring the wavelength of sound. Here sound waves, produced by a source S (such as a loudspeaker), travel in the air inside two different tubes. The sound waves then arrive at a detector D (such as a microphone) where the intensity of the resulting sound can be measured. The left tube, SBD, has some fixed length, while the length of the right tube SCD can be varied like that of a trombone. Suppose that, as the right tube is gradually lengthened by pulling the point C of this tube 1.70 cm to the right, the intensity observed at the detector D increases from a minimum value to a maximum value. (a) What then is the wavelength of the sound produced by the source S? (b) What is the frequency of this sound if the speed of sound in air is 340 m/s? *(Answer: 25) (Suggestion: [s-17])*

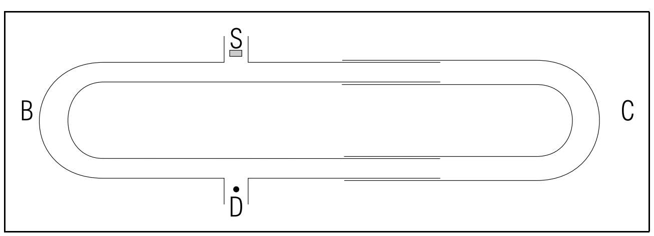


Fig. G-1.

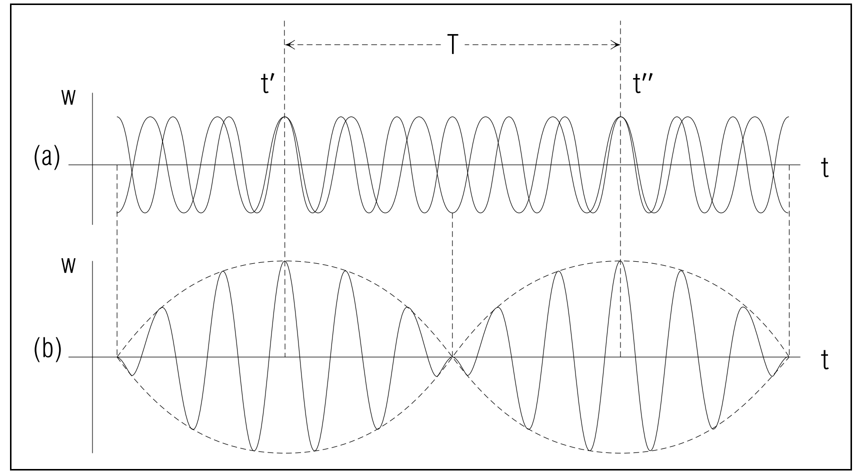


Fig. G-2.

G-4 *Interference of waves of different frequencies (beats):* Suppose that two sinusoidal waves at the same point have slightly different frequencies ν_1 and ν_2, as illustrated in Fig. G-2a. If the waves are in phase and thus interfere constructively at some particular time t', they will then not remain in phase because of their different frequencies. Hence they will later be out of phase and thus interfere destructively, and still later again be in phase and thus again interfere constructively. The resultant wave, shown in Fig. G-2b, behaves then like a sinusoidal wave with an amplitude varying repetitively with time. In the case of sound waves, such a *slowly* varying amplitude is perceived as a slow repetitive variation of the intensity of the individual waves, a phenomenon called "beats." (a) To find the time T elapsed between successive maxima of the resultant amplitude, note that this time must be such that, if the two individual waves both have their maximum value at the same time t', the wave with the larger frequency ν_2 repeats itself during the time T precisely once more often than the wave with the smaller frequency ν_1 (so that both of these waves again have their maximum values at the later time t" = t' + T).

By expressing this relation in terms of an equation, express T in terms of the frequencies ν_1 and ν_2. (b) The "beat frequency" ν_b is defined as the number of successive repetitions per unit time of the maximum amplitude. How is this beat frequency ν_b related to T? How then is ν_b related to the frequencies ν_1 and ν_2 of the individual waves? *(Answer: 20) (Suggestion: [s-19])*

G-5 *Beats used for tuning:* Beats can be very useful for tuning musical instruments. For example, suppose that a piano tuner has just tuned one of the several A strings of a piano so that it produces a tone with the standard frequency of 440 Hz. When sounding this A string at the same time as another one of the A strings, he hears the intensity varying with a beat frequency of 2.0 intensity maxima per second. He also notes that this beat frequency *decreases* when the frequency of this other string is increased (i.e., when this other string is stretched more tightly). On the basis of this information and the results of the preceding problem G-4, what is the frequency of the tone produced by this other string? *(Answer: 27) (Suggestion: [s-21])*

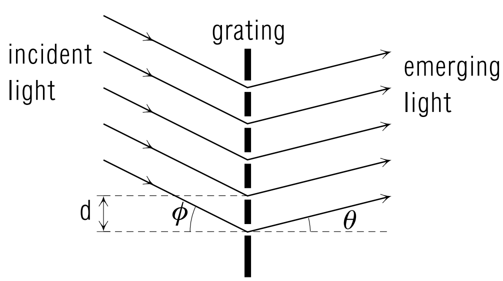
Note: Tutorial section G contains additional problems.

TUTORIAL FOR G

AdditionalProblems

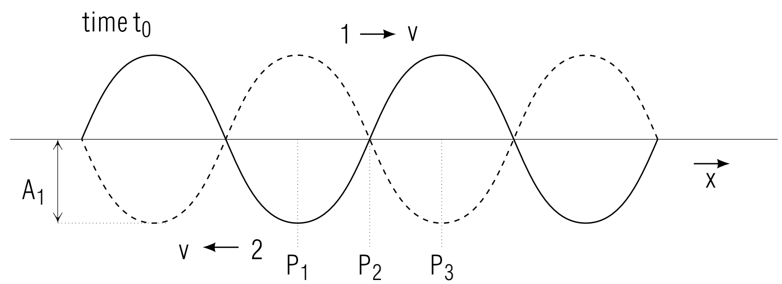
g-1 *SHARPNESS OF INTERFERENCE MAXIMA FROM A GRATING:* Consider light emanating in phase from the N regularly spaced slits of a grating. In this situation, constructive interference produces a sharp intensity maximum in a very narrow angular range $\Delta\theta$ where the waves from all the N slits arrive nearly in phase. Such neighboring sharp interference maxima are then separated from each other by some angle $\Delta\theta$. (a) Suppose that the waves did *not* interfere, so that the intensity due to light from all the N sources would be the *same* at all angles. What then would be this intensity, expressed in terms of the intensity I_1 of waves from a single slit? (b) Because of conservation of energy, the effect of the interference is to concentrate the total power, which would emerge into an angular range $\Delta\theta$ in the absence of interference, into the narrow angle range $\delta\theta$ of a single interference maximum of intensity $N^2 I_1$. Express this fact in terms of an equation. (c) What then is the ratio $\delta\theta/\Delta\theta$ of the angular width $\delta\theta$ of an interference maximum compared to the angular separation $\Delta\theta$ between two such neighboring interference maxima? (d) Suppose that the number N of slits in the grating were 3 times as large, the separation d between the slits remaining the same. How would this affect the angular separation $\Delta\theta$ between adjacent interference maxima? How would this affect the angular width $\delta\theta$ of *each* interference maximum? *(Answer: 63)*

g-2 *LIGHT OBLIQUELY INCIDENT ON A GRATING:* Figure H-1 indicates light from a distant source incident on a grating at an angle ϕ with respect to the grating.



(Thus the incident wavefronts make an angle ϕ with respect to the grating.) What then is the condition that an interference maximum should, in n^{th} order, be produced at large distances from the grating at an angle θ from the perpendicular to the grating? *(Answer: 62)*

g-3 *INTERFERENCE OF OPPOSITELY MOVING WAVES:* Two waves, with the same wavelength λ and the same amplitude A_1, move with the same speed V in opposite directions, the first wave along $\hat{x}$ and the second wave opposite to $\hat{x}$. (See the diagram.)



At a particular point P_1, the first wave attains its maximum value at the same time t_0 as the second wave attains its minimum value, so that the resultant wave is zero at this time. (a) At this point P_1, what is the resultant wave at the times $T/4$, $T/2$, $3T/4$, and T after the time t_0 (where T is the period of the waves)? What then is the amplitude of the resultant wave at this point P_1? (b) Consider now another point P_2 at a distance $\lambda/4$ to the right of the point P_1. What is the resultant wave at P_2 at the time t_0? What then is the amplitude of the resultant wave at this point P_2? (c) Consider now another point P_3 at a distance $\lambda/2$ to the right of the point P_1. What is the resultant wave at P_3 at the time t_0 and at the times $T/4$, $T/2$, $3T/4$, and T after the time t_0? What then is the amplitude of the resultant wave at this point P_3? (d) On the basis of the answers to the preceding questions, what is the distance between adjacent points at which the resultant wave is always equal to zero? *(Answer: 61)* *(Suggestion: s-22)*

PRACTICE PROBLEMS

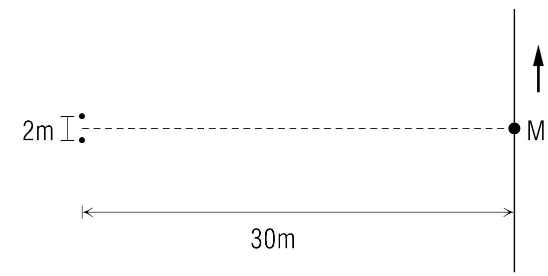
p-1 *RESULTANT AMPLITUDE & WAVE INTENSITY (CAP. 1):* Two radio waves of the same frequency arrive at the same point after being emitted by two different antennas. The electric fields of these two waves have at this point the same direction and are such that the amplitude of the electric field of the first wave is 3 volt/meter, while the amplitude of the electric field of the second wave is 2 volt/meter. (a) Express the intensity I_2 of the second wave in terms of the intensity I_1 of the first wave. (b) Suppose that the two waves are coherent and in phase. Express the intensity of the resultant wave in terms of I_1. (c) Suppose that the two waves are coherent and 1/2 cycle out of phase. Express the intensity of the resultant wave in terms of I_1. (d) Suppose that the two waves are incoherent. Express the intensity of the resultant wave in terms of I_1. How does this intensity compare with the average of the intensities found for the two extreme cases in parts *b* and *c*? *(Answer: 52) (Suggestion: Review text problem A-1.)*

p-2 *RESULTANT AMPLITUDE & INTENSITY WAVES (CAP. 1):* (a) Two coherent waves of the same frequency have at a given point amplitudes A_1 and A_2 whose ratio $A_1/A_2 = 5/2$. What then is the ratio $I_{\rm max}/I_{\rm min}$ of the maximum intensity of the resultant wave produced when both of these waves are in phase, as compared to the minimum intensity $I_{\rm min}$ of the resultant wave produced when both of these waves are 1/2 cycle out of phase? (b) Suppose that the ratio of the amplitudes of these waves is $A_1/A_2 = 2/5$ (i.e., the reciprocal of the ratio in part (a)). What then would be the ratio $I_{\rm max}/I_{\rm min}$? *(Answer: 55) (Suggestion: See [s-4] and review text problem A-1.)*

p-3 *RESULTANT AMPLITUDE & INTENSITY WAVES (CAP. 1):* When two coherent waves are in phase so as to interfere constructively, the intensity of the resultant wave is $I_{\rm max}$. When these waves are 1/2 cycle out of phase so as to interfere destructively, the intensity of the resultant wave is $I_{\rm min}$. Suppose that one observes experimentally that the ratio $I_{\rm max}/I_{\rm min} = 49$. Which of the following possibilities is then the ratio A_1/A_2 of the amplitudes of the two waves: 3/1, 3/2, 4/3, 5/3? *(Answer: 51) (Suggestion: Review the previous frame, [p-2].)*

p-4 *INTERFERENCE DUE TO TWO SOURCES (CAP. 2):* Two loudspeakers, separated by a distance of 2.0 m, are connected to the same

amplifier and emit in phase sound waves with a frequency of 680 Hz. The speed of sound in air is 340 m/s. As indicated in the diagram, a man starts walking along a line parallel to the speakers, at a distance of 30 m from the speakers, starting at a point M equidistant from the speakers. Thus the man hears at M sound of maximum intensity.



(a) How far along the line must the man walk before he hears the next sound of maximum intensity? (b) How many maxima intensity of sound can the man possibly hear (after the one at M) if he keeps on walking along the line in the same direction? *(Answer: 54) (Suggestion: See [s-10] and review text problem C-2.)*

p-5 *INTERFERENCE DUE TO TWO SOURCES (CAP. 2):* Interference of light from two slits: Light, having a wavelength of 5.0×10^{-7} m, is incident perpendicularly on a pair of very narrow slits separated by a distance of 0.10 mm. What then is the separation between the adjacent intensity maxima, observed because of interference, on an observation screen 0.40 m behind the two slits? *(Answer: 57) (Suggestion: Review text problem D-2.)*

p-6 *INTERFERENCE DUE TO TWO SOURCES (CAP. 2):* Determination of wavelength from double-slit experiment: In the double slit experiment illustrated in Fig. D-2 of the text, the narrow parallel slits are 2.0×10^{-4} m apart. The distance between adjacent interference intensity maxima, observed on a screen at a distance of 3.00 m behind the slits, is found to be 8.1×10^{-3} m. What then is the wavelength of the light passing through the slits? *(Answer: 59) (Suggestion: Review text problem D-2.)*

p-7 *INTERFERENCE DUE TO REGULARLY SPACED SOURCES (CAP. 3):* Positions of interference produced by a grating: Light, emanating from hydrogen atoms in an electric discharge of hydrogen gas, is incident perpendicularly upon a diffraction grating having 5000 lines per centimeter. What are all the angles, other than the angle $\theta = 0$ corresponding to the incident direction, where one would observe the interference maxima of blue light, with a wavelength of 4.34×10^{-7} m, emitted by hydrogen atoms? *(Answer: 58) (Suggestion: Review text problems E-1 and E-2.)*

p-8 *INTERFERENCE DUE TO REGULARLY SPACED SOURCES (CAP. 3):* Measurement of wavelength: In the situation described in the preceding problem, [p-7], one also observes interference maxima due to red light, in first order at an angle of 19.2°. What then is the wavelength of the red light emitted by hydrogen atoms? *(Answer: 53) (Suggestion: Review text problems E-1 and E-2.)*

p-9 *INTERFERENCE DUE TO REGULARLY SPACED SOURCES (CAP. 3):* Maximum wavelength observable in a given order: Light is incident perpendicularly on a diffraction grating having 4000 lines per centimeter. What is the longest wavelength of the interference maximum which can be observed in 5th order? *(Answer: 60) (Suggestion: See [s-15].)*

SUGGESTIONS

s-1 *(Text problem A-2):* Part a: How is the amplitude A_1 of the sound wave from one speaker related to the amplitude A_2 of the sound wave from the other speaker? What then is the amplitude A of the resulting sound wave? Express A in terms of A_1. What then is the corresponding intensity of this resulting wave, expressed in terms of A_1 and then also in terms of the intensity $I_1 = 10^{-4}\,\mathrm{W}/m^2$?

Part b: Proceed as in part a.

Part c: For incoherent waves, how is the intensity of the resulting wave related to the intensities of the separate waves?

s-2 *(Text problem B-2):* The path difference determines the phase difference acquired by the waves because they travel through different distances. If the waves leave the sources with the *same* phase, then the phase difference between the waves at some point P is just equal to the phase difference acquired by the waves as a result of their path difference. But if the waves leave the sources with a phase difference of 1/2 cycle, the phase difference between the waves at some point P is equal to this phase difference *plus* the phase difference acquired by the waves as a result of their path difference. For example, suppose that the waves leaving the sources are 1/2 cycle out of phase and then travel the *same* distance to some point P. What then is the phase difference between the waves at this point P? Do the waves there interfere so as to produce a maximum or a minimum intensity of the resultant wave?

s-3 *(Text problem A-1):* Part b: Sketch the two waves. How is the amplitude of the resultant wave related to the amplitudes A_1 and A_2 of the individual waves? (Remember that the amplitude is the maximum *magnitude* attained by a sinusoidal wave during its cycle and must thus be positive.) Express this result in terms of A_1 by using the fact that $A_2 = 2A_1$. What then is the intensity of the resultant wave, expressed in terms of A_1? Use the answer to part (a) to express this result in terms of I_1.

Part d: Remember, that for *incoherent* waves, the resultant intensity is just equal to the sum of the *intensities* of the individual waves.

s-4 *(Practice problem [p-2]):* Part a: If the ratio $A_1/A_2 = 5/2$, one can write $A_1 = 5k$ and $A_2 = 2k$, where k is any conveniently shown number. One can then express all calculations in terms of k. This number

k will then cancel whenever one calculates any ratio, such as A_1/A_2 or $I_{\rm max}/I_{\rm min}$.

Part b: You can, of course, redo the calculation. More conveniently, note that all that has been done is to reverse the names of the waves. Thus the wave which was previously called wave 1 is now called wave 2, and vice versa. Should the result of simple renaming the waves affect how these waves combine to yield a resultant wave?

s-5 *(Text problem A-3):* In each case, are the waves produced coherent or incoherent? If coherent, are they in phase or not? The rest of the problem is then the same as problem A-2.

s-6 *(Text problem C-4):* Note that the path difference from the two antennas is 0 at the points P_1 or P_3, but $1/2\,\lambda$ at the point P_2 or P_4. If the waves interfere constructively at the points P_1 or P_3, they then interfere destructively at the points P_2 or P_4, and vice versa.

s-7 *(Text problem C-2):* Find the wavelength of the waves from a knowledge of their frequency and their speed in the air.

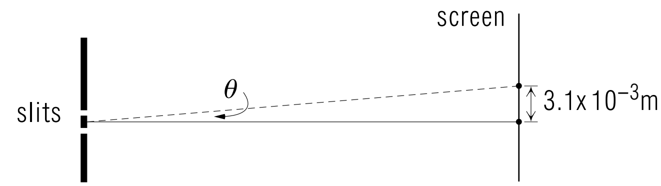
s-8 *(Text problem D-1):* Remember that blue light has a larger frequency (and thus a smaller wavelength) than red light. Is the angle for the first intensity maximum, after the central maximum at $\theta = 0$, larger for a larger or smaller wavelength?

s-9 *(Text problem B-3):* Compare each of the path differences with the wavelength $\lambda = 100\,{\rm m}$. Also note that $\sin 30° = 1/2$.

s-10 *(Practice problem [p-4]):* Find first the wavelength λ of the sound waves in air. Then find all the angles, measured from the central line, along which the intensity maxima occur. How many such angles are there, so that these angles intersect the line along which the man walks? The answer to this question provides the answer to part (b) of the problem. The distance along the line can then be found from the angle in the manner indicated in Fig. B-5 of the text.

s-11 *(Text problem F-2):* Consider the red light of the specified frequency. In water, where the speed of light is smaller than in vacuum, is the wavelength of this red light larger or smaller? Accordingly, for a fixed separation between the slits of the grating, is the angle at which interference occurs larger or smaller?

s-12 *(Text problem D-2):* On the basis of the given information, what is the angle between the central intensity maximum and the next intensity maximum on the observation screen? (See the diagram.) What then is the relation between θ, the separation d between the slits, and the wavelength λ?



Here, and in other problems where an angle θ is very small, calculations can be simplified by using the fact that, for small angles, $\sin\theta \approx \tan\theta \approx \theta$ (measured in radians).

s-13 *(Text problem F-1):* Part a: The resulting maximum intensity is related to the amplitudes of the waves so that $I_{\rm max} = \gamma(A_1 + A_2)^2$. If the intensity of source 2 is larger, the corresponding amplitude A_2 of the wave due to this source is also larger. Is $I_{\rm max}$ then larger or smaller?

Part b: The resulting minimum intensity is related to the amplitudes of the waves so that $I_{\rm min} = \gamma(A_1 - A_2)^2$. Suppose that the intensity of source 2 is made larger so that the amplitude A_2 of the second wave is also larger. What then happens to the intensity $I_{\rm min}$ if A_2 is originally smaller than A_1? What happens to $I_{\rm min}$ if A_2 is originally larger than A_1? [As specific examples, suppose that A_2 is originally negligibly small compared to A_1. What then happens to $I_{\rm min}$ if A_2 is made as large as A_1? Another specific example, suppose that originally $A_2 = A_1$ so that $I_{\rm min} = 0$. What then happens if A_2 is made larger than A_1?].

s-14 *(Text problem E-1):* If there are 4000 lines per centimeter, what is the separation d between adjacent slits of the grating? [s-15] (Practice problem [p-9])

s-15 *(Practice problem [p-9]):* Note that 90° is the maximum possible angle at which one can observe interference maxima from light emerging from the grating.

s-16 *(Text problem E-3):* Part c: Remember that the colors of the rainbow are (in order of increasing frequency) red, yellow, green, blue, and violet.

Part d: How is the resultant intensity produced by the simultaneous interference of N slits related to the intensity that would be produced by a *single* slit? What then happens to the resultant intensity if the grating is only half as wide so that it only has half as many slits?

s-17 *(Text problem G-3):* Part a: The waves emanating from S travel along the two tubes (i.e., along paths of different lengths) before they meet again at D. Originally the waves arriving at D are one-half cycle out of phase so as to interfere there destructively, while finally they arrive at D in phase so as to interfere there constructively. To produce this additional phase difference of one-half cycle, the path difference between the two waves must then have increased by $\lambda/2$ as a result of the lengthening of the right tube. But, when the point C of the right tube moves a distance L to the right, by what amount does the length of the right tube increase? (Note that this tube is bent so that it has both a top part and a bottom part.)

s-18 *(Text problem G-1):* First determine the relations between the amplitudes of all the waves, and then express these amplitudes in terms of the intensities of these waves.

s-19 *(Text problem G-4):* (a) If the second wave has a frequency ν_2, how many times does it repeat itself during the time T? (b) If the first waves has a frequency ν_1, how many times does it repeat itself during the time T? (c) What then is the equation expressing the fact that the number of repetitions of the second wave is larger than the number of repetitions of the first wave by one repetition? *(Answer 56)*

s-20 *(Text problem G-2):* Express the observed intensities in terms of the amplitudes of the individual waves. Then solve the resultant equations for the desired amplitudes.

s-21 *(Text problem G-5):* Using merely the fact that the beat frequency is 2.0 Hz, what are the two possible frequencies of the sound produced by the other string? Which of these two possibilities is correct if the beat frequency *decreases* if the frequency of the other string is increased?

s-22 *(Tutorial frame [g-3]):* For each of the several successive times sketch a diagram, similar to that in the problem, showing the waves after they have moved in opposite directions since the time t_0. Remember that

each wave moves a distance $\lambda/4$ during a time $T/4$.

ANSWERS TO PROBLEMS

1. a. min
   b. min
   c. max
   d. max
2. a. $4 \times 10^{-4}\,\mathrm{W/m^2}$
   b. 0
   c. $2 \times 10^{-4}\,\mathrm{W/m^2}$
3. a. max
   b. max
   c. min
   d. min
4. a. $I_2 = 4I_1$
   b. $A_{\rm max} = 3A_1$, $I_{\rm max} = 9I_1$
   c. $A_{\rm min} = A_1$, $I_{\rm min} = I_1$
   d. $I_{\rm inc} = 5I_1$
   e. $5I_1$, equal
5. a. 0
   b. 100 m
   c. 50 m
   d. max
   e. I (same as at P_{12})
   f. I (same as at P_{12})
   g. 0
   h. 0 (same as P_1)
6. a. $\pm 30°$, $\pm 90°$
   b. $\pm 14.5°$, $\pm 48.6°$
7. a. $4 \times 10^{-5}\,\mathrm{W/m^2}$
   b. $2 \times 10^{-5}\,\mathrm{W/m^2}$
8. 248 Hz

9. a. $\sin^{-1}(\lambda/2d)$
   b. larger
   c. 2 d
   d. The intensity becomes nowhere as small as $I_{\rm min}$
   e. smaller
10. a. larger
    b. larger
    c. smaller
11. a. 0, $\pm L(\lambda/d)$
    b. $\pm\frac{1}{2}L(\lambda/d)$
12. larger for red
13. a. $\pm 8.1°$
    b. $\pm 16.5°$
    c. $\pm 8.1°$
    d. same
    e. larger
14. a. $4I$, 0
    b. 0, $4I$
15. a. 6.2×10^3 Å
    b. 4.8×10^{14} Hz
16. smaller
17. 5.8×10^{-7} m
18. a. larger
    b. either larger or smaller (For explanation, see [s-13])
19. a. 9.2°, 16.3°; 7.1°
    b. 18.7°, 34.1°; 15.4°
    c. 28.7°, 57.1°; 28.4°
    d. no
    e. yes
20. a. $T = 1/(\nu_2 - \nu_1)$

b. $\nu_b = 1/T = \nu_2 - \nu_1$

21. a. $(I_{\rm max}^{1/2} - I_{\rm min}^{1/2})/(I_{\rm max}^{1/2} + I_{\rm min}^{1/2})$
    b. $\frac{1}{4}(I_{\rm max} + I_{\rm min} - 2\sqrt{I_{\rm max} I_{\rm min}})$
    c. $\frac{1}{4}(I_{\rm max} + I_{\rm min} + 2\sqrt{I_{\rm max} I_{\rm min}})$
22. a. 2.40×10^{-6} m
    b. 1.06×10^4
    c. smaller
    d. equal, $I = I_0/4$
23. BLK
24. a. $I_{\rm max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$
    b. $I_{\rm min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$
25. a. 6.80 cm
    b. 5.00×10^3 Hz
26. BLK
27. 438 Hz
28. BLK
29. BLK
51. 4/3
52. a. $(4/9)I_1$
    b. $(25/9)I_1$
    c. $(1/9)I_1$
    d. $(13/9)I_1$, equal
53. 6.57×10^{-7} m
54. a. 7.8 m
    b. 3
55. a. 49/9
    b. same answer
56. a. $\nu_2 T$

b. $\nu_1 T$
c. $\nu_2 T - \nu_1 T = 1$

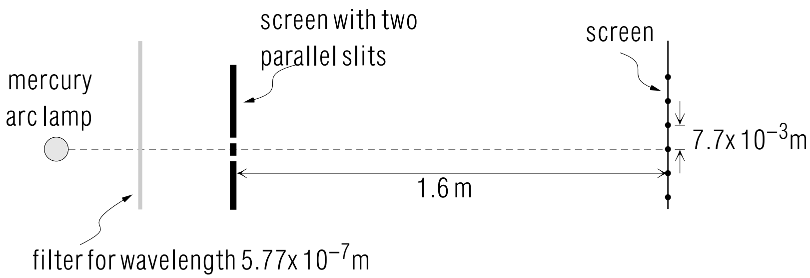
57. 2.0 mm
58. $\pm 12.5°$, $\pm 25.7°$, $\pm 40.6°$, $\pm 60.2°$
59. 5.4×10^{-7} m
60. 5.0×10^{-7} m
61. a. 0, 0, 0, 0
    b. 0, $2A_1$, 0, $-2A_1$
    c. 0, 0, 0, 0
    d. $\lambda/2$
62. $d(\sin\phi + \sin\theta) = n\lambda$
63. a. NI_1
    b. $(NI_1)\Delta\theta = (N^2 I_1)\delta\theta$
    c. $\delta\theta/\Delta\theta = 1/N$
    d. unchanged, 3 times smaller

MODEL EXAM

1. **Combination of two sound waves.** A sound wave of displacement amplitude 2.0×10^{-7} meter has an intensity of 3.0×10^{-3} watt/meter^2. If two such sound waves combine at a point in space at which they are one-half cycle out of phase, what is the intensity of the resulting wave?

2. **Some experiments with visible light.**

   The following diagram shows an arrangement for observing interference effects with the light from a mercury arc. A filter eliminates all light except that with a wavelength of 5.77×10^{-7} meter.



   a. Two parallel slits are placed in the filtered light beam. On a screen 1.6 meter away an interference pattern appears, with bright lines spaced 7.7×10^{-3} meter apart. What is the separation between the two slits?

   b. At the position of the first bright line away from the central line, what relationship exists between the distances from this line to the two slits?

   c. If the two slits were illuminated by separate, incoherent sources, what would be the nature of the pattern observed on the screen?

   d. Consider what would happen if a grating with 100 slits, spaced the same as the two slits previously considered, were used instead. How would the locations of the bright lines on the screen be changed? By what factor would the intensity at any point in one of the bright lines change? How would the appearance of the lines change other than in brightness?

Brief Answers:

1. Zero (or 0)
2. a. 1.2×10^{-4} meter

   b. $\Delta r = \pm\lambda$, or $r_2 - r_1$ is one wavelength

   c. no bright lines (more or less uniform intensity all over)

   d. locations: unchanged
   intensity: 2500 times greater
   other changes: lines narrower (sharper)